8-6 The Law of Sines and Law of Cosines

Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. 

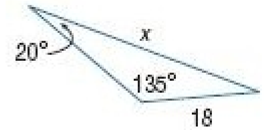
SOLUTION:

We are given the measure of two angles and a nonincluded side, so use the Law of Sines to write a proportion:

$$\begin{aligned} \frac{\sin 35}{x} &= \frac{\sin 58}{9} && \text{Substitute.} \\ x \cdot \sin 58 &= 9 \cdot \sin 35 && \text{Cross Products Property} \\ x &= \frac{9 \sin 35}{\sin 58} && \text{Divide each side by } \sin 58 \\ x &\approx 6.1 && \text{Use a calculator} \end{aligned}$$

ANSWER:

6.1

2. 

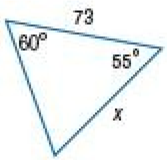
SOLUTION:

We are given two angles and a nonincluded side, therefore use Law of Sines to write a proportion:

$$\begin{aligned} \frac{\sin 135}{x} &= \frac{\sin 20}{18} && \text{Substitute.} \\ x \cdot \sin 20 &= 18 \cdot \sin 135 && \text{Cross Products Property} \\ x &= \frac{18 \sin 135}{\sin 20} && \text{Divide each side by } \sin 20 \\ x &\approx 37.2 && \text{Use a calculator} \end{aligned}$$

ANSWER:

37.2

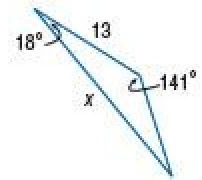
3. 

SOLUTION:
We are given one side and two angles, therefore use Law of Sines to write a proportion:

The angle opposite the side of length 73 has a degree measure of 180° – 60° – 55° = 65°.

$$
\begin{aligned}
\frac{\sin 60}{x} &= \frac{\sin 65}{73} && \text{Substitute.} \\
x(\sin 65) &= 73 \cdot (\sin 60) && \text{Cross Products Property} \\
x &= \frac{73(\sin 60)}{\sin 65} && \text{Divide each side by } \sin 65 \\
x &\approx 69.8 && \text{Use a calculator}
\end{aligned}
$$

ANSWER:
69.8

4. 

SOLUTION:
We are given one side and two angles so you can use Law of Sines to write a proportion.

The angle measure opposite the side of length 13 is: 180° – 141° – 18° = 21°.

$$
\begin{aligned}
\frac{\sin 141}{x} &= \frac{\sin 21}{13} && \text{Substitute.} \\
x \cdot \sin 21 &= 13 \cdot \sin 141 && \text{Cross Products Property} \\
x &= \frac{13 \sin 141}{\sin 21} && \text{Divide each side by } \sin 21 \\
x &\approx 22.8 && \text{Use a calculator}
\end{aligned}
$$

ANSWER:
22.8

5. 

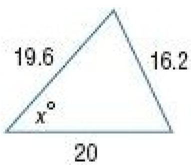
SOLUTION:

We are given the measures of two sides and their included angle, therefore we can use the Law of Cosines to solve this problem.

$$\begin{aligned} x^2 &= (14.7)^2 + (7)^2 - 2(14.7)(7)\cos(18) && \text{Substitute.} \\ x^2 &= 216.09 + 49 - 195.73 && \text{Simplify} \\ x^2 &= 69.36 && \text{Combine} \\ x &\approx 21 && \text{Take the square root of each side} \end{aligned}$$

ANSWER:

8.3

6. 

SOLUTION:

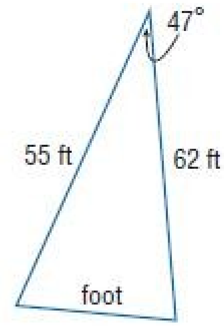
Since we are given three sides and no angles, we can use the Law of Cosines to solve for x.

$$\begin{aligned} (16.2)^2 &= (19.6)^2 + (20)^2 - 2(19.6)(20)\cos(x) && \text{Substitute.} \\ 262.44 &= 384.16 + 400 - 784\cos(x) && \text{Simplify} \\ 0.66546 &\approx \cos(x) && \text{Solve for } \cos(x) \\ 48 &\approx x && \text{Use the inverse cosine ratio} \end{aligned}$$

ANSWER:

48

7. **SAILING** Determine the length of the bottom edge, or foot, of the sail. Refer to photo on page 586.



SOLUTION:

Since we are given two sides and the included angle, we can solve this problem using the Law of Cosines. Let x represent the foot of the sail.

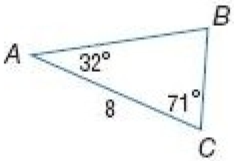
$$\begin{aligned} x^2 &= (55)^2 + (62)^2 - 2(55)(62)\cos(47) && \text{Substitute.} \\ x^2 &= 2217.77 && \text{Simplify} \\ x &= \sqrt{2217.77} && \text{Take the square root of each side} \\ x &\approx 47.1 && \text{Simplify} \end{aligned}$$

Therefore, the foot of the sail is about 47.1 ft long.

ANSWER:

47.1 ft

Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

8. 

SOLUTION:

The sum of the angles of a triangle is 180. So, $m\angle B = 180 - (32 + 71) = 77$.

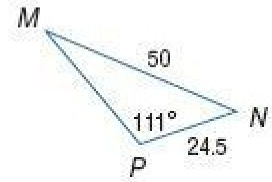
Since we are given two angles and a side, we can set up a proportion using the Law of Sines to find BC and AB.

$$\begin{array}{rcll} \frac{\sin 32}{BC} & = & \frac{\sin 77}{8} & \text{Substitute.} \\ BC \cdot \sin 77 & = & 8 \cdot \sin 32 & \text{Cross Products Property} \\ BC & = & \frac{8 \sin 32}{\sin 77} & \text{Divide each side by } \sin 77 \\ BC & \approx & 4.4 & \text{Use a calculator} \end{array}$$

$$\begin{array}{rcll} \frac{\sin 77}{8} & = & \frac{\sin 71}{AB} & \text{Substitute.} \\ AB \cdot \sin 77 & = & 8 \cdot \sin 71 & \text{Cross Products Property} \\ AB & = & \frac{8 \cdot \sin 71}{\sin 77} & \text{Divide each side by } \sin 77 \\ x & \approx & 7.8 & \text{Use a calculator} \end{array}$$

ANSWER:

$m\angle B = 77°, AB \approx 7.8, BC \approx 4.4$

9. 

SOLUTION:

Since we are given two sides and a nonincluded angle, we can set up a proportion using the Law of Sines to solve for the measure of $\angle M$.

$$\begin{array}{rcll} \frac{\sin M}{24.5} & = & \frac{\sin 111}{50} & \text{Substitute.} \\ (\sin M) \cdot 50 & = & 24.5 \cdot \sin 111 & \text{Cross Products Property} \\ \sin M & = & \frac{24.5 \sin 111}{50} & \text{Divide each side by } 50 \\ m\angle M & \approx & 27 & \text{Use a calculator} \end{array}$$

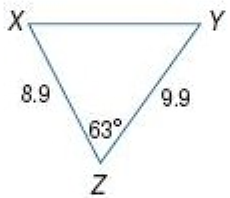
The sum of the angles of a triangle is 180. So, $m\angle N = 180 - (111 + 27) = 42$.

Now , we have two sides and an included angle, so we can use the Law of Cosines to solve for MP.

$$\begin{array}{rcll} (MP)^2 & = & (50)^2 + (24.5)^2 - 2(50)(24.5)\cos(42) & \text{Substitute.} \\ (MP)^2 & = & 1279.55 & \text{Simplify} \\ MP & = & \sqrt{1279.55} & \text{Take the square root of each side} \\ MP & \approx & 35.8 & \text{Simplify} \end{array}$$

ANSWER:

$m\angle M \approx 27°, m\angle N \approx 42°, MP \approx 35.8$

10. 

SOLUTION:

Since we are given two sides and an included angle, we can solve for *XY* using the Law of Cosines.

$$\begin{aligned} XY^2 &= (8.9)^2 + (9.9)^2 - 2(8.9)(9.9)\cos(63) && \text{Substitute.} \\ XY^2 &= 97.22 && \text{Simplify} \\ XY &= \sqrt{97.22} && \text{Take the square root of each side} \\ XY &\approx 9.9 && \text{Simplify} \end{aligned}$$

Since, ΔXYZ is (approximately) isosceles, the base angles are congruent. Therefore $m\angle X \approx 63°$ and $m\angle Y \approx 180 - (63 + 63) = 54.$

ANSWER:

$m\angle X \approx 63°$, $m\angle Y \approx 54°$, $XY \approx 9.9$

11. Solve ΔDEF if $DE = 16$, $EF = 21.6$, $FD = 20$.

SOLUTION:

Since we are given three sides and no angles, we can start solving this triangle by finding the $m\angle E$ by using the Law of Cosines.

$$\begin{aligned} (20)^2 &= (16)^2 + (21.6)^2 - 2(16)(21.6)\cos E && \text{Substitute.} \\ 400 &= 256 + 466.56 - 691.2\cos E && \text{Simplify} \\ 3.8087 &\approx \cos E && \text{Use the inverse cosine ratio} \\ 62 &\approx E && \text{Simplify} \end{aligned}$$

Similarly, we can use the Law of Cosines to solve for $m\angle D$.

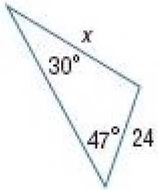
$$\begin{aligned} (21.6)^2 &= (16)^2 + (20)^2 - 2(16)(20)\cos D && \text{Substitute.} \\ 466.56 &= 256 + 400 - 640\cos D && \text{Simplify} \\ 0.296 &\approx \cos D && \text{Use the inverse cosine ratio} \\ 73 &\approx D && \text{Simplify} \end{aligned}$$

The sum of the angles of a triangle is 180. So,
$m\angle F = 180 - (73 + 62) = 45.$

ANSWER:

$m\angle D = 73°$, $m\angle E = 62°$, $m\angle F = 45°$

Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth.

12. 

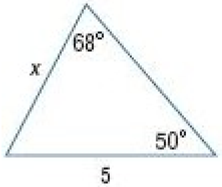
SOLUTION:

Since you are given two angles and a nonincluded side, you can set up a proportion using the Law of Sines.

$$\begin{aligned} \frac{\sin 47}{x} &= \frac{\sin 30}{24} && \text{Substitute.} \\ x \cdot \sin 30 &= 24 \cdot \sin 47 && \text{Cross Products Property} \\ x &= \frac{24 \sin 47}{\sin 30} && \text{Divide each side by } \sin 30 \\ x &\approx 35.1 && \text{Use a calculator} \end{aligned}$$

ANSWER:

35.1

13. 

SOLUTION:

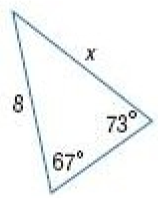
Since you are given two angles and a nonincluded side, you can set up a proportion using the Law of Sines.

$$\begin{aligned} \frac{\sin 50}{x} &= \frac{\sin 68}{5} && \text{Substitute.} \\ x \cdot \sin 68 &= 5 \cdot \sin 50 && \text{Cross Products Property} \\ x &= \frac{5 \sin 50}{\sin 68} && \text{Divide each side by } \sin 68 \\ x &\approx 4.1 && \text{Use a calculator} \end{aligned}$$

ANSWER:

4.1

8-6 The Law of Sines and Law of Cosines

14. 

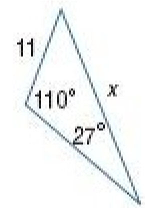
SOLUTION:

Since you are given two angles and a nonincluded side, you can set up a proportion using the Law of Sines.

$$\begin{aligned} \frac{\sin 67}{x} &= \frac{\sin 73}{8} && \text{Substitute.} \\ x \cdot \sin 73 &= 8 \cdot \sin 67 && \text{Cross Products Property} \\ x &= \frac{8\sin 67}{\sin 73} && \text{Divide each side by } \sin 73 \\ x &\approx 7.7 && \text{Use a calculator} \end{aligned}$$

ANSWER:

7.7



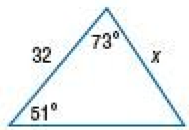
SOLUTION:

Since you are given two angles and a nonincluded side, you can set up a proportion using the Law of Sines.

$$\begin{aligned} \frac{\sin 110}{x} &= \frac{\sin 27}{11} && \text{Substitute.} \\ x \cdot \sin 27 &= 11 \cdot \sin 110 && \text{Cross Products Property} \\ x &= \frac{11\sin 110}{\sin 27} && \text{Divide each side by } \sin 27 \\ x &\approx 22.8 && \text{Use a calculator} \end{aligned}$$

ANSWER:

22.8

16. 

SOLUTION:

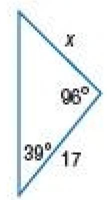
Since we are given one side and two angles, we can set up a proportion using the Law of Sines.

The measure of the angle opposite the side of length 32 is 180° – 73° – 51° = 56°.

$$\begin{aligned}
\frac{\sin 56}{32} &= \frac{\sin 51}{x} && \text{Substitute.}\\
x(\sin 56) &= 32 \cdot \sin 51 && \text{Cross Products Property}\\
x &= \frac{32 \sin 51}{\sin 56} && \text{Divide each side by 30}\\
x &\approx 30.0 && \text{Use a calculator}
\end{aligned}$$

ANSWER:

30.0

17. 

SOLUTION:

Since we are given one side and two angles, we can set up a proportion using the Law of Sines.

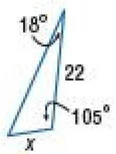
The angle opposite the side of length 17 has a degree measure of 180° – 96° – 39° = 45°.

$$\begin{aligned}
\frac{\sin 45}{17} &= \frac{\sin 39}{x} && \text{Substitute.}\\
x(\sin 45) &= 17 \cdot \sin 39 && \text{Cross Products Property}\\
x &= \frac{17 \sin 39}{\sin 45} && \text{Divide each side by 15}\\
x &\approx 15.1 && \text{Use a calculator}
\end{aligned}$$

ANSWER:

15.1

8-6 The Law of Sines and Law of Cosines

18. 

SOLUTION:

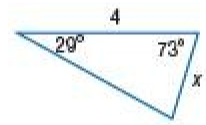
Since we are given one side and two angles, we can set up a proportion using the Law of Sines.

The measure of the angle opposite the side of length 22 is 180° – 105° – 18° = 57°

$$\begin{array}{rcll} \frac{\sin 57°}{22} & = & \frac{\sin 18}{x} & \text{Substitute.} \\ x(\sin 57) & = & 22 \cdot \sin 18 & \text{Cross Products Property} \\ x & = & \frac{22 \cdot \sin 18}{\sin 57} & \text{Divide each side by } \sin 57 \\ x & \approx & 8.1 & \text{Use a calculator} \end{array}$$

ANSWER:

8.1

19. 

SOLUTION:

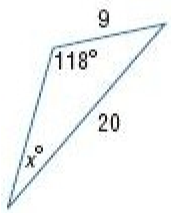
We are given one side and two angles, therefore we can use the Law of Sines to set up a proportion.

The angle opposite the side of length 4 has a degree measure of 180° – 29° – 73° = 78°

$$\begin{array}{rcll} \frac{\sin 29}{x} & = & \frac{\sin 78}{4} & \text{Substitute.} \\ (\sin 29) \cdot 4 & = & x \cdot \sin 78 & \text{Cross Products Property} \\ x & = & \frac{4 \cdot \sin 29}{\sin 78} & \text{Divide each side by } \sin 78 \\ x & \approx & 2.0 & \text{Use a calculator} \end{array}$$

ANSWER:

2.0

20. 

SOLUTION:

Since we are given one side and two angles, we can set up a proportion using the Law of Sines.

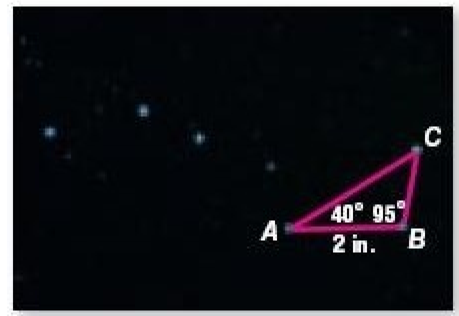
The angle opposite the side of length 12 has a degree measure of 180° – 118° – 39° = 23°.

$$\begin{aligned} \frac{\sin 23}{12} &= \frac{\sin 39}{x} && \text{Substitute.} \\ x \cdot \sin 23 &= 12 \cdot \sin 39 && \text{Cross Products Property} \\ x &= \frac{12 \cdot \sin 39}{\sin 23} && \text{Divide each side by } \sin 23 \\ x &\approx 19.3 && \text{Use a calculator} \end{aligned}$$

ANSWER:

19.3

21. **ASTRONOMY** Angelina is looking at the Big Dipper through a telescope. From her view, the cup of the constellation forms a triangle that has measurements shown on the diagram. Use the Law of Sines to determine distance between A and C.



SOLUTION:

The sum of the angles of a triangle is 180. So, $m\angle C = 180 - (40 + 95) = 45.$

Since we are given at least two angles and a side, we can set up a proportion using the Law of Sines.

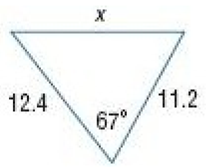
$$\begin{aligned} \frac{\sin 95}{x} &= \frac{\sin 45}{2} && \text{Substitute.} \\ x \cdot \sin 45 &= 2 \cdot \sin 95 && \text{Cross Products Property} \\ x &= \frac{2 \sin 95}{\sin 45} && \text{Divide each side by } \sin 45 \\ x &\approx 2.8 && \text{Use a calculator} \end{aligned}$$

Therefore, the distance between A and C is about 2.8 inches.

ANSWER:

2.8 in.

Find x. Round angle measures to the nearest degree and side measures to the nearest tenth.

22. 

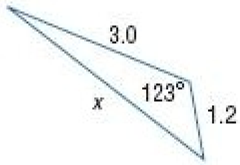
SOLUTION:

Since we have two sides and an included angle, we can use the Law of Cosines to solve for x.

x^2	$=$	$(12.4)^2+(11.2)^2-2(12.4)(11.2)\cos(67)$	Substitute.
x^2	$=$	170.67	Simplify
x	$=$	$\sqrt{170.67}$	Take the square root of each side
x	$\approx$	13.1	Simplify

ANSWER:

13.1

23. 

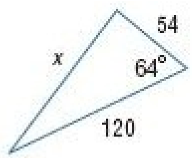
SOLUTION:

Since we are given two sides and an included angle, we can solve for the missing side using the Law of Cosines.

x^2	$=$	$(3.0)^2+(1.2)^2-2(3.0)(1.2)\cos(123)$	Substitute.
x^2	$=$	14.36	Simplify
x	$=$	$\sqrt{14.36}$	Take the square root of each side
x	$\approx$	3.8	Simplify

ANSWER:

3.8

24. 

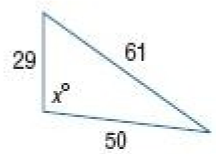
SOLUTION:

Since we are given two sides and an included angle, we can solve for x using the Law of Cosines.

x^2	$=$	$(120)^2+(54)^2-2(120)(54)\cos(64)$	Substitute.
x^2	$=$	11634.71	Simplify
x	$=$	$\sqrt{11634.71}$	Take the square root of each side
x	$\approx$	107.9	Simplify

ANSWER:

107.9

25. 

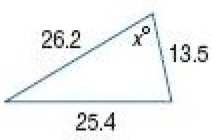
SOLUTION:

Since we are given three sides and no angles, we can find the measure of the missing angle by using the Law of Cosines.

$$\begin{aligned}(61)^2 &= (29)^2+(50)^2-2(29)(50)\cos(x) && \text{Substitute.}\\ 3721 &= 841+2500-2900\cos(x) && \text{Simplify}\\ -0.13103 &= \cos(x) && \text{Solve for } \cos(x)\\ x &\approx 98 && \text{Use the inverse cosine ratio}\end{aligned}$$

ANSWER:

98

26. 

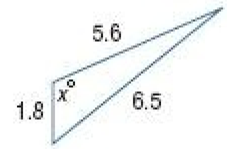
SOLUTION:

Since we given three sides and no angles, we can solve for the missing angle using the Law of Cosines.

$$\begin{aligned}(25.4)^2 &= (26.2)^2+(13.5)^2-2(26.2)(13.5)\cos(x) && \text{Substitute.}\\ 645.16 &= 686.44+182.25-707.4\cos(x) && \text{Simplify}\\ 0.3160 &= \cos(x) && \text{Solve for } \cos(x)\\ x &\approx 72 && \text{Use the inverse cosine ratio}\end{aligned}$$

ANSWER:

72

27. 

SOLUTION:

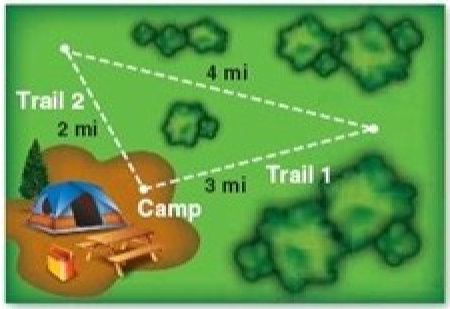
Since we are given three sides and no angles, we can solve for the missing angle using the Law of Cosines.

$$\begin{aligned}(6.5)^2 &= (1.8)^2+(5.6)^2-2(1.8)(5.6)\cos(x) && \text{Substitute.}\\ 42.25 &= 3.24+31.36-20.16\cos(x) && \text{Simplify}\\ -0.3795 &= \cos(x) && \text{Solve for } \cos(x)\\ x &\approx 112 && \text{Use the inverse cosine ratio}\end{aligned}$$

ANSWER:

112°

28. **HIKING** A group of friends who are camping decide to go on a hike. According to the map, what is the angle between Trail 1 and Trail 2?



SOLUTION:

Since we are given three side lengths and no angle measures, we can use the Law of Cosines to solve this problem. Let *x* be the angle between Trail 1 and Trail 2.

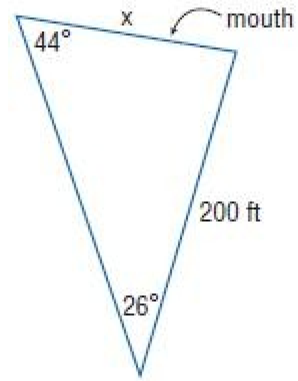
$$\begin{aligned} (4)^2 &= (3)^2 + (2)^2 - 2(3)(2)\cos(x) && \text{Substitute.} \\ 16 &= 9 + 4 - 12\cos(x) && \text{Simplify} \\ -0.25 &= \cos(x) && \text{Solve for } \cos(x) \\ x &\approx 104 && \text{Use the inverse cosine ratio} \end{aligned}$$

Therefore, the angle between Trail 1 and Trail 2 is about 104°.

ANSWER:

104°

29. **TORNADOES** Find the width of the mouth of the tornado. Refer to the photo on Page 588.



SOLUTION:

Since we are given two angles and a nonincluded side, we can use the Law of Sines to set up a proportion.

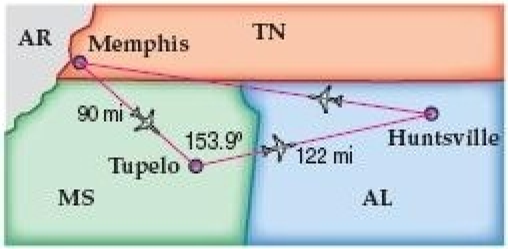
$$\begin{aligned} \frac{\sin 26}{x} &= \frac{\sin 44}{200} && \text{Substitute.} \\ x \cdot \sin 44 &= 200 \cdot \sin 26 && \text{Cross Products Property} \\ x &= \frac{200 \sin 26}{\sin 44} && \text{Divide each side by } \sin 44 \\ x &\approx 126.2 && \text{Use a calculator} \end{aligned}$$

Therefore, the mouth of the tornado is about 126.2 feet wide.

ANSWER:

126.2 ft

30. **TRAVEL** A pilot flies 90 miles from Memphis, Tennessee , to Tupelo, Mississippi, to Huntsville, Alabama, and finally back to Memphis. How far is Memphis from Huntsville?



SOLUTION:

Since we are given two sides and an included angle, we can use the Law of Cosines to solve this problem. Let x be the distance between Memphis and Huntsville.

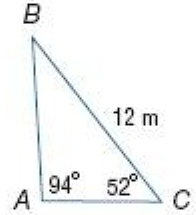
$$\begin{aligned} x^2 &= (90)^2 + (122)^2 - 2(90)(122)\cos(153.9) && \text{Substitute.} \\ x^2 &= 42704.69 && \text{Simplify} \\ x &= \sqrt{42704.69} && \text{Take the square root of each side} \\ x &\approx 207 && \text{Simplify} \end{aligned}$$

Therefore, Memphis is about 207 miles away from Huntsville.

ANSWER:

207 mi

Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

31. 

SOLUTION:

The sum of the angles of a triangle is 180. So,
$m\angle B = 180 - (94 + 52) = 34.$

Since we are given two sides and a nonincluded angle, we can set up a proportion using the Law of Sines to find AB.

$$\begin{aligned} \frac{\sin 52}{AB} &= \frac{\sin 94}{12} && \text{Substitute.} \\ AB \cdot \sin 94 &= 12 \cdot \sin 52 && \text{Cross Products Property} \\ AB &= \frac{12 \cdot \sin 52}{\sin 94} && \text{Divide each side by } \sin 94 \\ x &\approx 9.5 && \text{Use a calculator} \end{aligned}$$

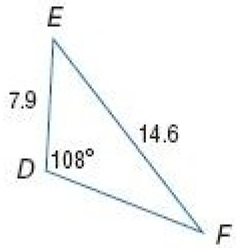
Similarly, we can use the Law of Sines to find CA.

$$\begin{aligned} \frac{\sin 34}{CA} &= \frac{\sin 94}{12} && \text{Substitute.} \\ CA \cdot \sin 94 &= 12 \cdot \sin 34 && \text{Cross Products Property} \\ CA &= \frac{12 \sin 34}{\sin 94} && \text{Divide each side by } \sin 94 \\ CA &\approx 6.7 && \text{Use a calculator} \end{aligned}$$

ANSWER:

$m\angle B = 34°, AB \approx 9.5, CA \approx 6.7$

CCSS STRUCTURE Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

32. 

SOLUTION:

Since we are given two sides and a nonincluded angle, we can find the measure of angle F with the Law of Sines.

$$\begin{aligned} \frac{\sin F}{7.9} &= \frac{\sin 108}{14.6} && \text{Substitute.} \\ 14.6 \cdot (\sin F) &= 7.9 \cdot (\sin 108) && \text{Cross Products Property} \\ \sin F &= \frac{7.9 \cdot \sin 108}{14.6} && \text{Divide each side by 14.6} \\ F &\approx 31 && \text{Use the inverse sine ratio} \end{aligned}$$

The sum of the angles of a triangle is 180. So, $m\angle E = 180 - (108 + 31) = 41$.

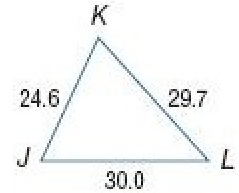
Since we are now know two sides and an included angle, we can use the Law of Cosines to find *DF*.

$$\begin{aligned} (DF)^2 &= 7.9^2 + 14.6^2 - 2(7.9)(14.6)\cos(41) && \text{Substitute.} \\ (DF)^2 &= 101.47 && \text{Simplify} \\ DF &= \sqrt{101.47} && \text{Take the square root of each side} \\ DF &\approx 10.1 && \text{Use a calculator} \end{aligned}$$

ANSWER:

$m\angle E = 41°$, $m\angle F = 31°$, $DF \approx 10.1$

Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

33. 

SOLUTION:

Since we are given all three sides of the triangle and no angles, we can find a missing angle using the Law of Cosines.

$$\begin{aligned} 30.0^2 &= (24.6)^2 + (29.7)^2 - 2(24.6)(29.7)\cos K && \text{Substitute.} \\ 900 &= 605.16 + 882.09 - 1461.24\cos K && \text{Simplify} \\ 0.40188 &= \cos K && \text{Solve for cos K} \\ K &\approx 66 && \text{Use the inverse cosine ratio} \end{aligned}$$

Similarly, we can use the Law of Cosines to find the measure of angle J.

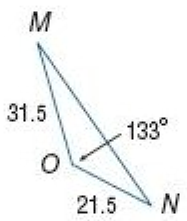
$$\begin{aligned} 29.7^2 &= (30)^2 + (24.6)^2 - 2(30)(24.6)\cos J && \text{Substitute.} \\ 882.09 &= 900 + 605.16 - 1476\cos J && \text{Simplify} \\ 0.42213 &= \cos J && \text{Solve for cos J} \\ J &\approx 65 && \text{Use the inverse cosine ratio} \end{aligned}$$

The sum of the angles of a triangle is 180. So, $m\angle L = 180 - (66 + 65) = 49$.

ANSWER:

$m\angle J \approx 65°$, $m\angle K \approx 66°$, $m\angle L \approx 49°$

8-6 The Law of Sines and Law of Cosines

34. 

SOLUTION:

Since we are given two sides and an included angle, we can find *MN* using the Law of Cosines.

$$
\begin{aligned}
MN^2 &= (31.5)^2 + (21.5)^2 - 2(31.5)(21.5)\cos 133 && \text{Substitute.}\\
MN^2 &= 992.25 + 462.25 + 923.77 && \text{Simplify}\\
MN^2 &= 2378.27 && \text{Combine}\\
MN &\approx 48.8 && \text{Take the square root of each side}
\end{aligned}
$$

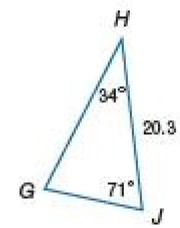
Since we have two sides and a nonincluded angle, we can use the Law of Sines to find the measure of angle M.

$$
\begin{aligned}
\frac{\sin 133}{48.8} &= \frac{\sin M}{21.5} && \text{Substitute.}\\
\sin M \cdot 48.8 &= \sin 133 \cdot 21.5 && \text{Cross Products Property}\\
\sin M &= \frac{\sin 133 \cdot 21.5}{48.8} && \text{Solve for sinM}\\
M &\approx 19 && \text{Use the inverse sine ratio}
\end{aligned}
$$

The sum of the angles of a triangle is 180. So, $m\angle N = 180 - (133 + 19) = 28$.

ANSWER:

$m\angle M \approx 19°$, $m\angle N \approx 28°$, $MN \approx 48.8$

35. 

SOLUTION:

Since we are given two sides and a nonincluded angle, we can find the measure of angle *J* using the Law of Sines.

$$
\begin{aligned}
\frac{\sin J}{20} &= \frac{\sin 34}{11.8} && \text{Substitute.}\\
\sin J \cdot 11.8 &= \sin 34 \cdot 20 && \text{Cross Products Property}\\
\sin J &= \frac{\sin 34 \cdot 20}{11.8} && \text{Solve for } \sin J\\
J &\approx 71 && \text{Use the inverse sine ratio}
\end{aligned}
$$

The sum of the measures of the angles of a triangle is 180. Therefore, $m\angle G = 180 - (34 + 71) = 75$.

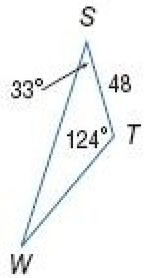
We can find *HJ*, using the Law of Sines, since we have two sides and a nonincluded angle.

$$
\begin{aligned}
\frac{\sin 75}{HJ} &= \frac{\sin 34}{11.8} && \text{Substitute.}\\
HJ \cdot \sin 34 &= \sin 75 \cdot 11.8 && \text{Cross Products Property}\\
HJ &= \frac{\sin 75 \cdot 11.8}{\sin 34} && \text{Solve for } HJ\\
HJ &\approx 20.3 && \text{Use your calculator.}
\end{aligned}
$$

ANSWER:

$m\angle G \approx 75°$, $m\angle J \approx 71°$, $HJ \approx 20.3$

8-6 The Law of Sines and Law of Cosines

36. 

SOLUTION:

The sum of the angles of a triangle is 180. So,
$m\angle W = 180 - (33 + 124) = 23.$

Since we have two angles and a nonincluded side, we can use the Law of Sines to find the measure of *TW*.

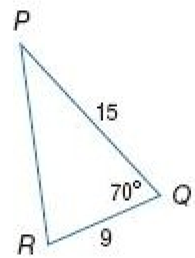
$$\begin{aligned} \frac{\sin 33}{WT} &= \frac{\sin 23}{48} && \text{Substitute.} \\ WT \cdot \sin 23 &= 48 \cdot \sin 33 && \text{Cross Products Property} \\ WT &= \frac{48 \cdot \sin 33}{\sin 23} && \text{Divide each side by } \sin 23 \\ WT &\approx 66.9 && \text{Use a calculator} \end{aligned}$$

Similarly, we can use the Law of Sines to find *WS*.

$$\begin{aligned} \frac{\sin 124}{WS} &= \frac{\sin 23}{48} && \text{Substitute.} \\ WS \cdot \sin 23 &= 48 \cdot \sin 124 && \text{Cross Products Property} \\ WS &= \frac{48 \cdot \sin 124}{\sin 23} && \text{Divide each side by } \sin 23 \\ WS &\approx 101.8 && \text{Use a calculator} \end{aligned}$$

ANSWER:

$m\angle W \approx 23°$, $WS \approx 101.8$, $TW \approx 66.9$

37. 

SOLUTION:

We are given two sides and an included angle, therefore we can use the Law of Cosines to solve for *PR*.

$$\begin{aligned} (PR)^2 &= 9^2 + 15^2 - 2(9)(15)\cos(70) && \text{Substitute.} \\ (PR)^2 &= 213.65 && \text{Simplify} \\ PR &= \sqrt{216.65} && \text{Take the square root of each side} \\ PR &\approx 14.6 && \text{Use a calculator} \end{aligned}$$

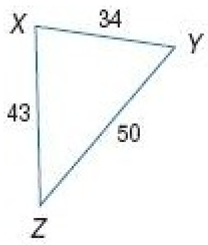
Now that we have two sides and a nonincluded angle, we can use the Law of Sines to find the measure of angle *R*.

$$\begin{aligned} \frac{\sin 70}{14.6} &= \frac{\sin R}{15} && \text{Substitute.} \\ 14.6 \cdot \sin R &= 15 \cdot \sin 70 && \text{Cross Products Property} \\ \sin R &= \frac{15 \cdot \sin 70}{14.6} && \text{Divide each side by } 14.6 \\ R &\approx 75 && \text{Use the inverse sine ratio} \end{aligned}$$

The sum of the angles of a triangle is 180. So,
$m\angle P = 180 - (75 + 70) = 35.$

ANSWER:

$m\angle P \approx 35°$, $m\angle R \approx 75°$, $RP \approx 14.6$

38. 

SOLUTION:

Since we are given three sides of a triangle and no angles, we can find a missing angle measure using the Law of Cosines.

$$\begin{aligned} (34)^2 &= 43^2+50^2-2(43)(50)\cos(Z) && \text{Substitute.} \\ 1156 &= 1849+2500-4300\cos Z && \text{Simplify} \\ 0.742558 &\approx \cos Z && \text{Solve for } \cos Z \\ Z &\approx 42 && \text{Use the inverse cosine ratio} \end{aligned}$$

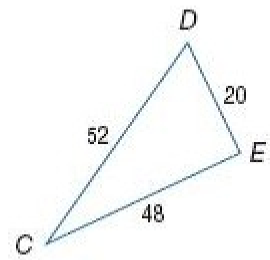
Similarly, we can use the Law of Cosines to find the measure of angle *Y*.

$$\begin{aligned} (43)^2 &= 34^2+50^2-2(34)(50)\cos Y && \text{Substitute.} \\ 1849 &= 1156+2500-3400\cos Y && \text{Simplify} \\ 0.53147 &\approx \cos Z && \text{Solve for } \cos Z \\ Z &\approx 58 && \text{Use the inverse cosine ratio} \end{aligned}$$

The sum of the angles of a triangle is 180. So,
$m\angle X = 180-(42+58)=80.$

ANSWER:

$m\angle X \approx 80°, m\angle Y \approx 58°, m\angle Z \approx 42°$

39. 

SOLUTION:

We are given three sides and no angles, therefore we can use the Law of Cosines to solve for the measure of a missing angle.

$$\begin{aligned} 52^2 &= 20^2+48^2-2(20)(48)\cos(E) && \text{Substitute.} \\ 2704 &= 400+2304-1920\cos E && \text{Simplify} \\ 0 &= \cos E && \text{Solve for } \cos E \\ E &\approx 90 && \text{Use the inverse cosine ratio} \end{aligned}$$

Similarly, we can use the Law of Cosines to find the measure of angle D.

$$\begin{aligned} 48^2 &= 20^2+52^2-2(20)(52)\cos(D) && \text{Substitute.} \\ 2304 &= 400+2704-2080\cos D && \text{Simplify} \\ 0.3846 &= \cos D && \text{Solve for } \cos D \\ D &\approx 67 && \text{Use the inverse cosine ratio} \end{aligned}$$

The sum of the angles of a triangle is 180. So,
$m\angle C = 180-(90+67)=23.$

ANSWER:

$m\angle C \approx 23°, m\angle D \approx 67°, m\angle E \approx 90°$

40. Solve ΔJKL if $JK = 33$, $KL = 56$, $LJ = 65$.

SOLUTION:

We are given three sides and no angles, therefore we can use the Law of Cosines to solve for the measure of a missing angle.

$$\begin{aligned} 65^2 &= 56^2 + 33^2 - 2(56)(33)\cos(K) && \text{Substitute.} \\ 4225 &= 3136 + 1089 - 3696\cos K && \text{Simplify} \\ 0 &= \cos K && \text{Solve for } \cos K \\ K &\approx 90 && \text{Use the inverse cosine ratio} \end{aligned}$$

Similarly, we can use the Law of Cosines to find the measure of angle *J*.

$$\begin{aligned} 56^2 &= 33^2 + 65^2 - 2(33)(65)\cos(J) && \text{Substitute.} \\ 3136 &= 1089 + 4225 - 4290\cos J && \text{Simplify} \\ 0.5077 &= \cos J && \text{Solve for } \cos J \\ J &\approx 59 && \text{Use the inverse cosine ratio} \end{aligned}$$

The sum of the angles of a triangle is 180. So,
$m\angle L = 180 - (90 + 59) = 31$.

ANSWER:

$m\angle L \approx 31$, $m\angle K \approx 90$, $m\angle J \approx 59$

41. Solve ΔABC if $m\angle B = 119$, $m\angle C = 26$, $CA = 15$.

SOLUTION:

The sum of the angles of a triangle is 180. So,
$m\angle W = 180 - (119 + 26) = 35$.

Since we are given two sides and a nonincluded angle, we can set up a proportion using the Law of Sines to find *AB*.

$$\begin{aligned} \frac{\sin 26}{AB} &= \frac{\sin 119}{15} && \text{Substitute.} \\ AB \cdot \sin 119 &= 15 \cdot \sin 26 && \text{Cross Products Property} \\ AB &= \frac{15 \cdot \sin 26}{\sin 119} && \text{Divide each side by } \sin 119 \\ AB &\approx 7.5 && \text{Use a calculator} \end{aligned}$$

Similarly, we can use the Law of Sines to find *CB*.

$$\begin{aligned} \frac{\sin 35}{BC} &= \frac{\sin 119}{15} && \text{Substitute.} \\ BC \cdot \sin 119 &= 15 \cdot \sin 35 && \text{Cross Products Property} \\ BC &= \frac{15 \sin 35}{\sin 119} && \text{Divide each side by } \sin 119 \\ BC &\approx 9.8 && \text{Use a calculator} \end{aligned}$$

ANSWER:

$m\angle A = 35$, $AB \approx 7.5$, $BC \approx 9.8$

42. Solve ΔXYZ if $XY = 190$, $YZ = 184$, $ZX = 75$.

SOLUTION:

Since we are given all three sides of the triangle and no angles, we can find a missing angle using the Law of Cosines.

75^2	$=$	$(190)^2 + (184)^2 - 2(190)(184)\cos Y$	Substitute.
5625	$=$	$36100 + 33856 - 69920\cos Y$	Simplify
0.9201	$=$	$\cos Y$	Solve for $\cos Y$
Y	$\approx$	23	Use the inverse cosine ratio

Similarly, we can use the Law of Cosines to find the measure of angle X.

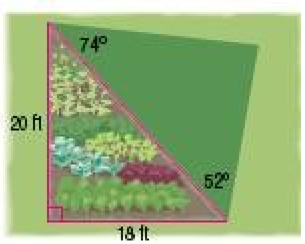
184^2	$=$	$(75)^2 + (190)^2 - 2(75)(190)\cos X$	Substitute.
33856	$=$	$5625 + 36100 - 28500\cos X$	Simplify
0.27611	$=$	$\cos X$	Solve for $\cos X$
X	$\approx$	74	Use the inverse cosine ratio

The sum of the angles of a triangle is 180. So,
$m\angle Z = 180 - (23 + 74) = 83$.

ANSWER:

$m\angle X \approx 74$, $m\angle Y \approx 23$, $m\angle Z \approx 83$

43. **GARDENING** Crystal has an organic vegetable garden. She wants to add another triangular section so that she can start growing tomatoes. If the garden and neighboring space have the dimensions shown, find the perimeter of the new garden.

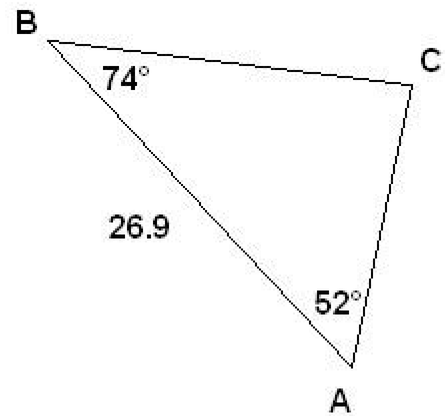


SOLUTION:

Use the Pythagorean Theorem to find the length of the hypotenuse of the right triangle.

$$18^2 + 20^2 = x^2$$
$$324 + 400 = x^2$$
$$724 = x^2$$
$$x \approx 26.9$$

We have the following triangle:



$m\angle C = 180° - 52° - 74° = 54°$.

Since we are given one side and two angles, we can find BC with the Law of Sines.

$\frac{\sin 52}{BC}$	$=$	$\frac{\sin 54}{26.9}$	Substitute.
$BC \cdot (\sin 54)$	$=$	$26.9 \cdot (\sin 52)$	Cross Products Property
BC	$=$	$\frac{26.9 \cdot \sin 52}{\sin 54}$	Divide each side by $\sin 54$
BC	$\approx$	26.2	Use a calculator

We can use the Law of Cosines to find AC, since we know two sides and an included angle.

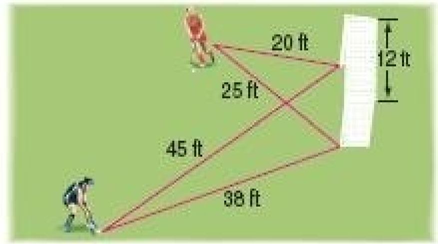
$$\begin{aligned}(AC)^2 &= 26.2^2+26.9^2-2(26.2)(26.9)\cos(74) && \text{Substitute.}\\ (AC)^2 &= 1022 && \text{Simplify}\\ AC &= \sqrt{1022} && \text{Take the square root of each side}\\ AC &\approx 32.0 && \text{Use a calculator}\end{aligned}$$

Therefore, the perimeter of the garden will be about 18 + 20 + 26.2 + 32.0 = 96.2 ft.

ANSWER:

96.2 ft

44. **FIELD HOCKEY** Alyssa and Nari are playing field hockey. Alyssa is standing 20 feet from one post of the goal and 25 feet from the opposite post. Nari is standing 45 feet from one post of the goal and 38 feet from the other post. If the goal is 12 feet wide, which player has a greater chance to make a shot? What is the measure of the player's angle?



SOLUTION:

Since Alyssa is closer to the goal, so she has a greater chance to make a goal.

Let x be the measure of Alyssa's angle with the two ends of the post. We can use the Law of Cosines to find Alyssa's shooting angle, since we have three sides of the triangle.

$$\begin{aligned}(12)^2 &= 20^2+25^2-2(20)(25)\cos(x) && \text{Substitute.}\\ 144 &= 400+625-1000\cos(x) && \text{Simplify}\\ 0.881 &= \cos(x) && \text{Solve for } \cos(x)\\ x &\approx 28.2 && \text{Use the inverse cosine ratio}\end{aligned}$$

Therefore, Alyssa's angle is about 28.2º.

ANSWER:

Alyssa; 28.2

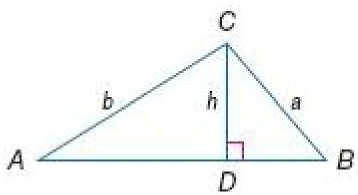
PROOF Justify each statement for the derivation of the Law of Sines.

45. **Given:** $\overline{CD}$ is an altitude of ΔABC .

Prove: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Proof:

Statements	Reasons
a. $\sin A = \frac{h}{b}, \sin B = \frac{h}{a}$	**a.** ___?___
b. $b \sin A = h, a \sin B = h$	**b.** ___?___
c. $b \sin A = a \sin B$	**c.** ___?___
d. $\frac{\sin A}{a} = \frac{\sin B}{b}$	**d.** ___?___



SOLUTION:

a. Def. of sine

b. Mult. Prop.

c. Subs.

d. Div. Prop.

ANSWER:

a. Def. of sine

b. Mult. Prop.

c. Subs.

d. Div. Prop.

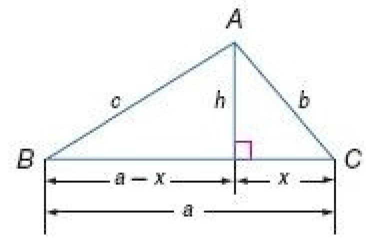
PROOF Justify each statement for the derivation of the Law of Cosines.

46. **Given:** h is an altitude of ΔABC .

Prove: $c2 = a2 + b2 – 2\ ab \cos C$

Proof:

Statements	Reasons
a. $c^2 = (a - x)^2 + h^2$	**a.** ___?___
b. $c^2 = a^2 - 2ax + x^2 + h^2$	**b.** ___?___
c. $x^2 + h^2 = b^2$	**c.** ___?___
d. $c^2 = a^2 - 2ax + b^2$	**d.** ___?___
e. $\cos C = \frac{x}{b}$	**e.** ___?___
f. $b \cos C = x$	**f.** ___?___
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	**g.** ___?___
h. $c^2 = a^2 + b^2 - 2ab \cos C$	**h.** ___?___



SOLUTION:

a. Pythagorean Thm.

b. Subs.

c. Pythagorean Thm.

d. Subs.

e. Def. of cosine

f. Mult. Prop

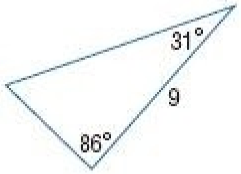
g. Subs.

h. Comm. Prop.

ANSWER:

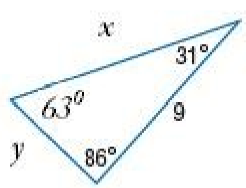
a. Pythagorean Thm.
b. Subs.
c. Pythagorean Thm.
d. Subs.
e. Def. of cosine
f. Mult. Prop
g. Subs.
h. Comm. Prop.

Find the perimeter of each figure. Round to the nearest tenth.

47. 

SOLUTION:
The sum of the angles of a triangle is 180. So, the measure of the third angle is 180 – (31 + 86) = 63.

We now have the following triangle:



We can use the Law of Sines to find the lengths of the missing sides, since we have two angles and the nonincluded side.

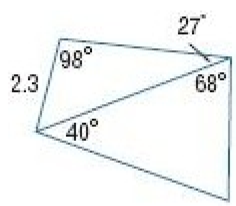
$$\frac{\sin 63°}{9} = \frac{\sin 86°}{x}$$
$$x \cdot \sin 63 = 9 \cdot \sin 86$$
$$x = \frac{9 \cdot \sin 86}{\sin 63}$$
$$x = 10.1$$

Solve for y, using the Law of Sines:

$$\frac{\sin 63°}{9} = \frac{\sin 31°}{y}$$
$$y \cdot \sin 63 = 9 \cdot \sin 31$$
$$y = \frac{9 \cdot \sin 31}{\sin 63}$$
$$y = 5.2$$

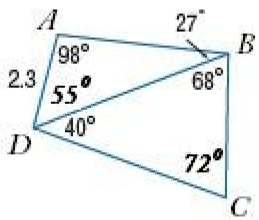
Therefore, the perimeter of the triangle is about 9 + 10.1 + 5.2 = 24.3 units.

ANSWER:
24.3

48. 

SOLUTION:
Name the quadrilateral *ABCD*, as shown. The sum of the angles of a triangle is 180. So,

$m\angle ADB = 180 - (98 + 27) = 55$ and $m\angle C = 180 - (68 + 40) = 72$.



We can use the Law of Sines to find the length of the missing sides,

$$\frac{\sin 27°}{2.3} = \frac{\sin 55°}{AB}$$
$$AB \cdot \sin 27 = \sin 55 \cdot 2.3$$
$$AB = \frac{\sin 55 \cdot 2.3}{\sin 27}$$
$$AB = 4.1$$

$$\frac{\sin 27°}{2.3} = \frac{\sin 98°}{BD}$$
$$BD \cdot \sin 27 = 2.3 \cdot \sin 98$$
$$BD = \frac{2.3 \cdot \sin 98}{\sin 27}$$
$$BD = 5.0$$

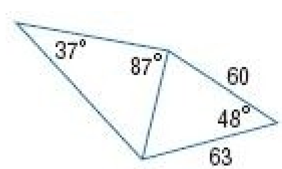
$$\frac{\sin 72°}{5} = \frac{\sin 40°}{BC}$$
$$BC \cdot \sin 72 = 5 \cdot \sin 40$$
$$BC = \frac{5 \cdot \sin 40}{\sin 72}$$
$$BC = 3.4$$

$$\frac{\sin 72°}{5} = \frac{\sin 68°}{DC}$$
$$DC \cdot \sin 72 = 5 \cdot \sin 68$$
$$DC = \frac{5 \cdot \sin 68}{\sin 72}$$
$$DC = 4.9$$

Therefore, the perimeter of the quadrilateral is about 2.3 + 4.1 +3.4 + 4.9 = 14.7 units.

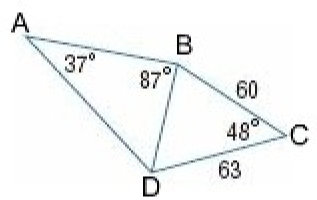
ANSWER:

14.7

49. 

SOLUTION:

Name the quadrilateral as *ABCD*.



We can find *BD* using the Law of Cosines because we know two sides and an included angle.

$$(BD)^2 = (60)^2 + (63)^2 - 2(60)(63)\cos 48°$$
$$(BD)^2 = 3600 + 3969 - 5058.6$$
$$(BD)^2 = 2510.4$$
$$BD \approx 50.1$$

The sum of the angles of a triangle is 180. So, $m\angle ADB = 180 - (37 + 87) = 56.$

We can use the Law of Sines to solve for *AD* and *AB*, since we have two angles and a nonincluded side.

$$\frac{\sin 37°}{50.1} = \frac{\sin 87°}{AD}$$
$$AD \cdot \sin 37 = 50.1 \cdot \sin 87$$
$$AD = \frac{50.1 \cdot \sin 87}{\sin 37}$$
$$AD = 83.1$$

$$\frac{\sin 37°}{50.1} = \frac{\sin 56°}{AB}$$
$$AB \cdot \sin 37 = 50.1 \cdot \sin 56$$
$$AB = \frac{50.1 \cdot \sin 56}{\sin 37}$$
$$AB = 69.0$$

Therefore, the perimeter of the quadrilateral is about 63 + 60 +83.1 + 69.0 = 275.1 units.

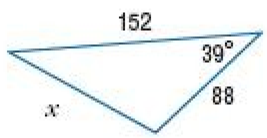
ANSWER:

275.1

50. 

SOLUTION:

Let *x* be the length of the third side of the triangle.



We can use the Law of Cosines to solve for *x*, since we have two sides and a nonincluded angle.

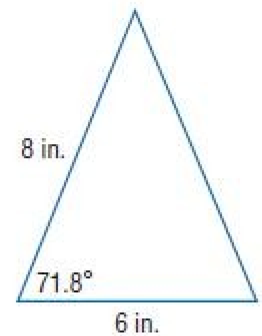
$$(x)^2 = (152)^2 + (88)^2 - 2(152)(88)\cos 39°$$
$$(x)^2 = 23104 + 7744 - 20790.2$$
$$(x)^2 = 10057.79$$
$$x \approx 100.3$$

Therefore, the perimeter is about 152 + 88 + 100.3 = 340.3 units.

ANSWER:

340.3

51. **MODELS** Vito is working on a model castle. Find the length of the missing side (in inches) using the model. Refer to Page 590.



SOLUTION:

Since we are given two sides and an included angle, we can use the Law of Cosines to solve this problem. Let *x* be the length of the missing side.

$$\begin{aligned} x^2 &= (6)^2 + (8)^2 - 2(6)(8)\cos(71.6) && \text{Substitute.} \\ x^2 &= 69.7 && \text{Simplify} \\ x &= \sqrt{69.7} && \text{Take the square root of each side} \\ x &\approx 8.4 && \text{Simplify} \end{aligned}$$

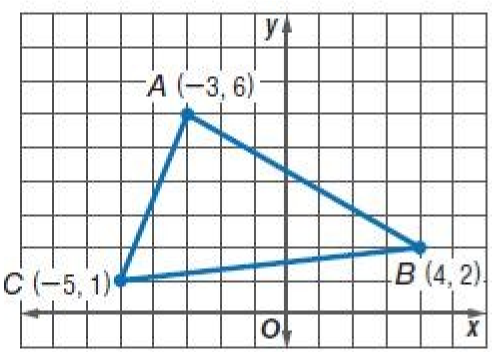
Therefore, the length of the missing side is about 8.4 inches.

ANSWER:

8.4 in.

52. **COORDINATE GEOMETRY** Find the measure of the largest angle in ΔABC with coordinates $A(-3, 6)$, $B(4, 2)$, and $C(-5, 1)$. Explain your reasoning.

SOLUTION:



Find the lengths of each segment of the triangle.

$AB = \sqrt{(4-(-3))^2 + (2-6)^2} \approx 8.1$

$BC = \sqrt{(-5-4)^2 + (1-6)^2} \approx 9.1$

$AB = \sqrt{(-5(-3))^2 + (1-6)^2} \approx 5.4$

9.1 > 8.1 > 5.4, so $BC > AB > AC$. Using the Triangle Inequality Theorem, the angle opposite the longest side, or $\angle A$, is the largest angle.

To find the measure of $\angle A$, use the Law of Cosines.

$(9.1)^2 = (8.1)^2 + (5.4)^2 - 2(8.1)(5.4)\cos A$

$82.81 = 65.61 + 29.16 - 87.48\cos A$

$0.1367 = \cos A$

$82.1 = A$

Therefore,

ANSWER:

82;

Sample answer: Find the lengths of each segment of the triangle.

$AB = \sqrt{(4-(-3))^2 + (2-6)^2} \approx 8.1$

$BC = \sqrt{(-5-4)^2 + (1-6)^2} \approx 9.1$

$AB = \sqrt{(-5(-3))^2 + (1-6)^2} \approx 5.4$

9.1 > 8.1 > 5.4, so $BC > AB > AC$. Using the Triangle Inequality Theorem, the angle opposite the longest side, or $\angle A$, is the largest angle. To find the measure of $\angle A$, use the Law of Cosines.

$(9.1)^2 = (8.1)^2 + (5.4)^2 - 2(8.1)(5.4)\cos A$

$m\angle A = 82$

53. **MULTIPLE REPRESENTATIONS** In this problem, you will use trigonometry to find the area of a triangle.

a. GEOMETRIC Draw an acute, scalene ΔABC including an altitude of length h originating at vertex A.

b. ALGEBRAIC Use trigonometry to represent h in terms of $m\angle B$.

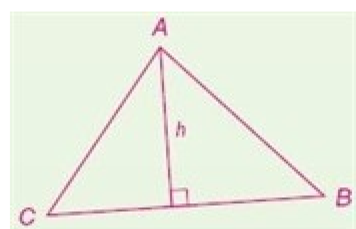
c. ALGEBRAIC Write an equation to find the area of ΔABC using trigonometry.

d. NUMERICAL If $m\angle B$ is 47, AB = 11.1, BC = 14.1, and CA = 10.4, find the area of ΔABC. Round to the nearest tenth.

e. ANALYTICAL Write an equation to find the area of ΔABC using trigonometry in terms of a different angle.

SOLUTION:

a. Label your triangle carefully, as described.



b. Use right triangle trigonometry to write an equation relating angle with side h.

By the definition of sine of an angle, $\sin B = \frac{h}{AB}$. So, $h = AB \sin B$.

c. The area of a triangle is given by the formula $A = \frac{1}{2}bh$. In this triangle, $b = BC$ and $h = AB \sin B$.

Therefore, $A = \frac{1}{2}(BC)(AB \sin B)$.

d. Substitute the given values in the area formula from part c.

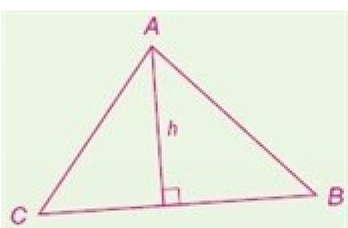
$$A = \frac{1}{2}(14.1)((11.1)\sin 47) \approx 57.2$$

e. Go through the process again, but choose angle C.

$$A = \frac{1}{2}(BC)(CA \sin C)$$

ANSWER:

a.



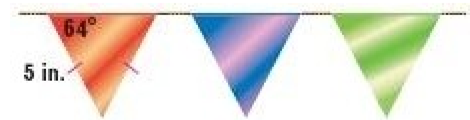
b. $h = AB \sin B$

c. $A = \frac{1}{2}(BC)(AB \sin B)$

d. 57.2 units^2

e. $A = \frac{1}{2}(BC)(CA \sin C)$

54. **ERROR ANALYSIS** Colleen and Mike are planning a party. Colleen wants to sew triangular decorations and needs to know the perimeter of one of the triangles to buy enough trim. The triangles are isosceles with angle measurements of $64°$ at the base and side lengths of 5 inches. Colleen thinks the perimeter is 15.7 inches and Mike thinks it is 15 inches. Is either of them correct?



SOLUTION:

The measure of the third angle of each triangle is 180 – (64 + 64) = 52.

Use the Law of Sines to find the length x of the base of each triangle.

$$\frac{\sin 64°}{5} = \frac{\sin 52°}{x}$$

$$x \cdot \sin 64 = 5 \cdot \sin 52$$

$$x = \frac{5 \cdot \sin 52}{\sin 64}$$

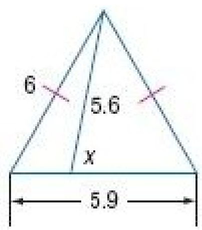
$$x \approx 4.4$$

The perimeter of each triangle is about 5 + 5 + 4.4 = 14.4 in. Therefore, neither is correct.

ANSWER:

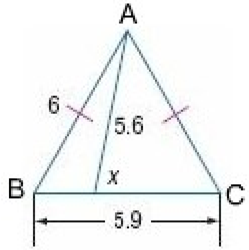
Neither; 5 + 5 + 4.4 = 14.4 in.

55. **CHALLENGE** Find the value of x in the figure.



SOLUTION:

Name the triangle as ΔABC.



We can find the measures of the base angles in isosceles triangle ABC, using the Law of Cosines.

$$\begin{aligned} 6^2 &= 6^2 + 5.9^2 - 2(6)(5.9)\cos C \\ 36 &= 36 + 34.81 - 70.8\cos C \\ -34.81 &= -70.8\cos C \\ 0.4917 &\approx \cos C \\ m\angle C &\approx 60.55 \end{aligned}$$

We can now use the Law of Sines with the smaller triangle on the right to solve for *x*.

$$\begin{aligned} \frac{\sin 60.55}{5.6} &= \frac{\sin x}{6} \\ 5.6 \cdot \sin x &= 6 \cdot \sin 60.55 \\ \sin x &= \frac{6 \cdot \sin 60.55}{5.6} \\ x &= 68.9 \end{aligned}$$

ANSWER:

$68.9°$

56. **REASONING** Are there any sets of three measures that cannot be used to solve a triangle? Explain.

SOLUTION:

When considering this answer, think of all the three possible angle and/or side combinations that you could be given. Are there any combinations that could result in triangle that is not congruent to another with the same three dimensions?

Yes; sample answer: If the three measures given are the angle measures, you cannot solve for the lengths of the sides. The triangle could be an infinite number of similar triangles.

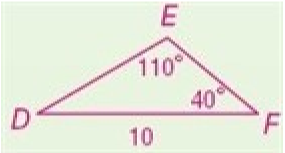
ANSWER:

Yes; sample answer: If the three measures given are the angle measures, you cannot solve for the lengths of the sides. The triangle could be an infinite number of similar triangles.

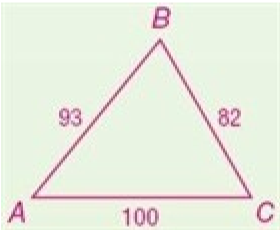
57. **OPEN ENDED** Draw and label a triangle that can be solved:
a. using only the Law of Sines.
b. using only the Law of Cosines.

SOLUTION:

a. Law of Sines can be used to solve a triangle when you are given two sides and a nonincluded angle or two angles and a nonincluded side.

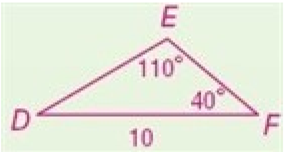


b. Law of Cosines can be used to solve a triangle when you are given three sides and no angles or two sides and an included angle.

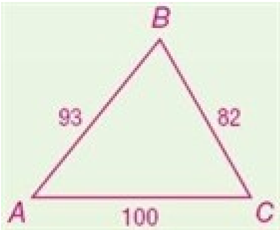


ANSWER:

a.

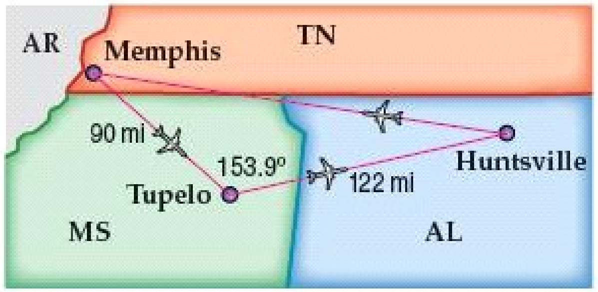


b.



58. **WRITING IN MATH** Suppose you use a calculator and determine the answer to Exercise 30 to be about 57 miles. Could this be correct? Explain your reasoning. What does this example tell your about the limitations of using calculators to solve problems?

30. TRAVEL A pilot flies 90 miles from Memphis, Tennessee, to Tupelo, Mississippi, to Huntsville, Alabama, and finally back to Memphis. How far is Memphis from Huntsville?



SOLUTION:

Use the Triangle Inequality Theorem when considering the answer to this question. Can you have a triangle with sides 57, 90, and 122?

This cannot be correct because the length of the indicated side of the triangle is not greater than the sum of the lengths of the other two sides. That is, $57 \ngtr 90 + 122$. This example illustrates that a calculator cannot detect human error. It is important to use estimation and mathematical reasoning to asses whether or not an answer given by a calculator is reasonable.

ANSWER:

This cannot be correct because the length of the indicated side of the triangle is not greater than the sum of the lengths of the other two sides. That is, $57 \ngtr 90 + 122$. This example illustrates that a calculator cannot detect human error. It is important to use estimation and mathematical reasoning to asses whether or not an answer given by a calculator is reasonable.

59. In ΔABC, if $m\angle A = 42°$, $m\angle B = 74°$, and $a = 3$, what is the value of b?
A 4.3
B 3.8
C 2.1
D 1.5

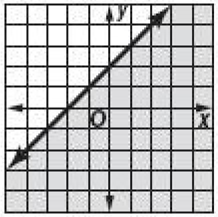
SOLUTION:
By the Law of Sines,

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 42°}{3} = \frac{\sin 74°}{b}$$
$$b \cdot \sin 42 = 3 \cdot \sin 74$$
$$b = \frac{3 \cdot \sin 74}{\sin 42}$$
$$b \approx 4.3$$

Therefore, the correct choice is A.

ANSWER:
A

60. **ALGEBRA** Which inequality best describes the graph below?



F $y \geq -x + 2$
G $y \leq x + 2$
H $y \geq -3x + 2$
J $y \leq 3x + 2$

SOLUTION:
The slope of the line is 1 and it intersects the y-axis at (0, 2), so the equation of the line is $y = x + 2$.

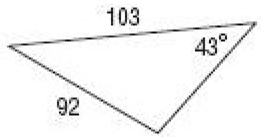
The region below the line is shaded, so the inequality is $<$ or $\leq$. Since the boundary is a solid line, the symbol is $\leq$.

Therefore, the equation of the graph is $y \leq x + 2$.

The correct choice is G.

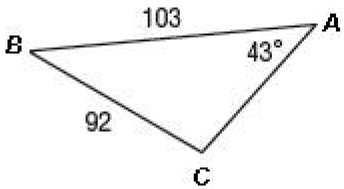
ANSWER:
G

61. **SHORT RESPONSE** What is the perimeter of the triangle shown below? Round to the nearest tenth.



SOLUTION:

Name the triangle as ABC.



By the Law of Sines, $\frac{\sin 43°}{92} = \frac{\sin C}{103}$.

Simplify.

$\sin C = \frac{103 \cdot \sin 43°}{92} \approx 0.7635$

Take the inverse and round it to the nearest degree.

$C \approx 49.8$

The sum of the angles of a triangle is 180. So, the measure of the third angle is 180 – (43 + 49.8) = 87.2.

By the Law of Cosines, $(BC)^2 = (92)^2 + (103)^2 - 2(92)(103)\cos 87.2°$.

Simplify.

$(BC)^2 \approx 18147.2$

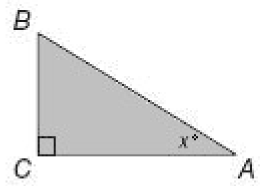
$BC \approx 134.7$

Therefore, the perimeter of the triangle is about 92 + 103 + 134.7 = 329.7 units.

ANSWER:

329.7

62. **SAT/ACT** If $\sin x = 0.6$ and $AB = 12$, what is the area of ΔABC?



A 9.6 units^2

B 28.8 units^2

C 31.2 units^2

D 34.6 units^2

E 42.3 units^2

SOLUTION:

$\sin x = \frac{BC}{AB}$

$0.6 = \frac{BC}{12}$

$BC = 7.2$

Use the Pythagorean Theorem to find AC.

$AC^2 + 7.2^2 = 12^2$

$AC^2 = 92.16$

$AC = 9.6$

The area of a triangle is given by the formula $A = \frac{1}{2}bh$.

$A = \frac{1}{2}(9.6)(7.2)$

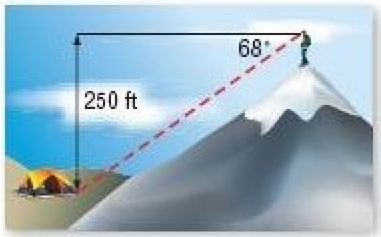
$A \approx 34.6$.

Therefore, the correct choice is D.

ANSWER:

D

63. **HIKING** A hiker is on top of a mountain 250 feet above sea level with a $68°$ angle of depression. She can see her camp from where she is standing. How far is her camp from the top of the mountain?



SOLUTION:

$$\sin A = \frac{\text{Opposite side}}{\text{Hypotenuse}}.$$

Let x be the distance between the camp and the hiker.

$$\sin 68° = \frac{250}{x}$$

$$x \cdot \sin 68 = 250$$

$$x = \frac{250}{\sin 68}$$

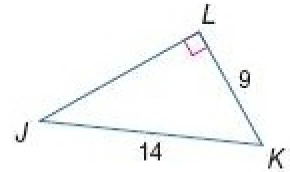
$$x = 269.6$$

Therefore, the hiker is about 269.6 ft away from the camp.

ANSWER:

269.6 ft

Use a calculator to find the measure of $\angle J$ to the nearest degree.

64. 

SOLUTION:

$$\sin A = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

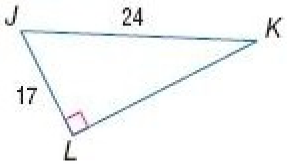
$$\sin J = \frac{9}{14}$$

$$J = \sin^{-1}\left(\frac{9}{14}\right)$$

$$J \approx 40$$

ANSWER:

40

65. 

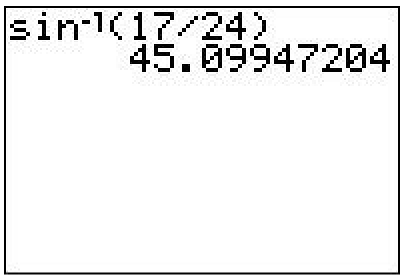
SOLUTION:

$$\sin K = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\sin K = \frac{17}{24}$$

$$K = \sin^{-1}\left(\frac{17}{24}\right)$$

$$K = 45$$



ANSWER:

45

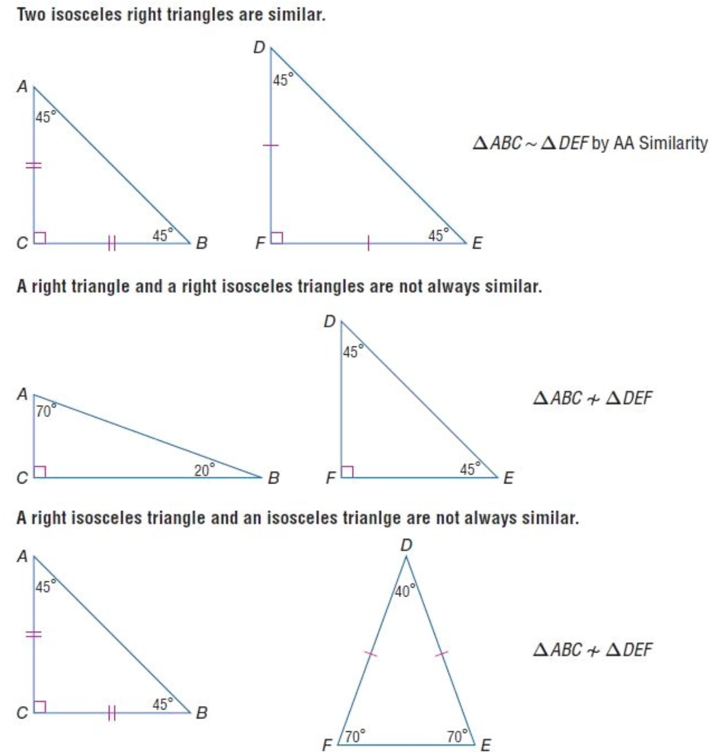
Determine whether the polygons are always, sometimes, or never similar. Explain your reasoning.

66. a right triangle and an isosceles triangle

SOLUTION:

Consider the properties of a right triangle and an isosceles triangle, as well as what it takes for triangles to be similar.

If corresponding angles are congruent and corresponding sides are proportional, a right triangle and an isosceles triangle are similar. Therefore, the statement is *sometimes* true.



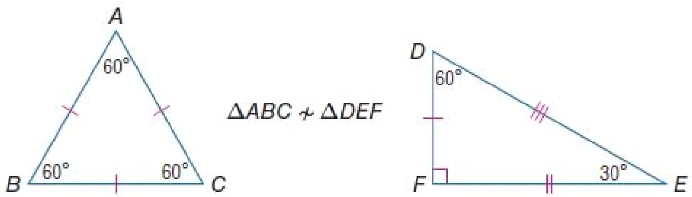
ANSWER:

Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, a right triangle and an isosceles triangle are similar.

67. an equilateral triangle and a scalene triangle

SOLUTION:

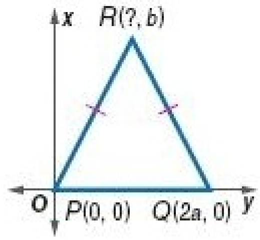
Never; sample answer: Since an equilateral triangle has three congruent sides and a scalene triangle has three non-congruent sides, the ratios of the three pairs of sides can never be equal. Therefore, an equilateral triangle and a scalene triangle can never be similar.



ANSWER:

Never; sample answer: Since an equilateral triangle has three congruent sides and a scalene triangle has three non-congruent sides, the ratios of the three pairs of sides can never be equal. Therefore, an equilateral triangle and a scalene triangle can never be similar.

Name the missing coordinates of each triangle.

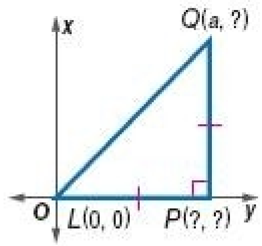
68. 

SOLUTION:

Since $\triangle PQR$ is isosceles the x-coordinate of R is located halfway between 0 and $2a$, so it is a. Therefore, the coordinates of the vertex R is (a, b).

ANSWER:

$R(a, b)$

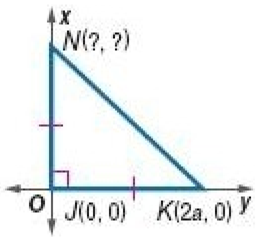
69. 

SOLUTION:

The vertex P is on the x-axis. So, the y-coordinate of the vertex P is zero. The x-coordinates of the vertices P and Q are same. Therefore the coordinates of the vertex P is $(a, 0)$.
Since $LP = PQ$ and the lines are vertical and horizontal respectively, the y-coordinate of Q will be the x-coordinate of P and it is a. Therefore, the coordinates of the vertex Q is (a, a).

ANSWER:

$Q(a, a)$ $P(a, 0)$

70. 

SOLUTION:

The vertex N is on the y-axis. So, the x-coordinate of the vertex N is zero. Since $JK = JN$ and the lines are vertical and horizontal respectively, the x-coordinate of K will be the y-coordinate of N and it is $2a$. Therefore, the coordinates of the vertex N is $(0, 2a)$.

ANSWER:

$N(0, 2a)$

Find the distance between each pair of points. Round to the nearest tenth.

71. $A(5, 1)$ and $C(-3, -3)$

SOLUTION:

Use the Distance Formula.

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} AC &= \sqrt{(-3-5)^2 + (-3-1)^2} \\ &= \sqrt{(-8)^2 + (-4)^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} \approx 8.9 \end{aligned}$$

The distance between A and C is about 8.9 units.

ANSWER:

$\sqrt{80} \approx 8.9$

72\. *J*(7, 11) and *K*(–1, 5)

SOLUTION:
Use the Distance Formula.

$$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} JK &= \sqrt{(-1-7)^2 + (5-11)^2} \\ &= \sqrt{(-8)^2 + (-6)^2} \\ &= \sqrt{64+36} \\ &= \sqrt{100} = 10 \end{aligned}$$

The distance between *J* and *K* is 10 units.

ANSWER:
10.0

73\. *W*(2, 0) and *X*(8, 6)

SOLUTION:
Use the Distance Formula.

$$WX = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} WX &= \sqrt{(8-2)^2 + (6-0)^2} \\ &= \sqrt{(6)^2 + (6)^2} \\ &= \sqrt{36+36} \\ &= \sqrt{72} \approx 8.5 \end{aligned}$$

The distance between *W* and *X* is about 8.5 units.

ANSWER:
$\sqrt{72} \approx 8.5$